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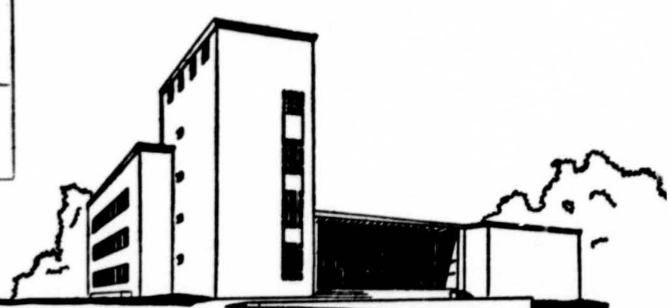

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TIME-DEPENDENT DELAYS
AT TRAFFIC MERGES

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Graduate School of Industrial Administration
Carnegie Institute of Technology
and
Westinghouse Research Laboratories

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INTRODUCTION

Analysis of the delay experienced by a side-road driver attempting to merge with, or cross, traffic on a main road has been conducted by many authors; see Bisbee and Oliver [2], Buckley and Blunden [3], Evans, Herman, and Weiss [4], Garwood [5], Gaver [6], Hawkes[8], Jewell[9], and others. Postulating various behavioral characteristics for side-road drivers, the above authors have presented formulas describing the long-run delays of drivers attempting to merge or cross, and other related measures such as side-road queue magnitudes.

The purpose of the present paper is to study one such model numerically, with special emphasis upon time-dependent behavior. For particular examples we shall exhibit the manner in which expected delay either converges to the long-run limit, if a finite limit exists, or grows if there is saturation. Without such information we are forced to rely upon the "steady-state" or long-run results of queueing theory. Steady-state values may be reached rather slowly under some conditions, e.g. during rush hours; it is perhaps useful to have some idea of the adequacy of the long-run predictions at various times after the start of the rush hour. One factor that emerges as important in this study is the effect of mixing "fast" with "slow" drivers in various proportions on the

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side-road, "fast" drivers being those willing to accept relatively short main-road gaps for merge or crossing, "slow drivers (e.g. possibly buses and trucks) requiring somewhat more time.

Finally, our numerical procedure allows the study of natural approximations to the queueing processes that arise in this, and other, contexts. We give an example of one such approximation that is related to diffusion theory and the "heavy traffic" approximations of J.F.C. Kingman. It seems sensible to attempt to find plausible and comprehensible approximations for the queueing processes arising in traffic, as in other areas of applied probability. Numerical methods provide one means of discovering and evaluating such approximations, and of uncovering important differences in the various models that have been proposed.

THE MODEL

Our basic model has been introduced in an earlier paper [6]. We shall assume that main road traffic is a stationary Poisson process with rate ν and that main road vehicles can be considered to be points. Side-road or merging traffic is likewise Poisson with rate λ. There are two types of side-road drivers: Type I will be called, for want of a better term "fast" and Type 2 will be called "slow", although other adjectives may be more meaningful; the restriction to two types is easily lifted. The probability that fast and slow drivers appear at the merger of the roads in any order is that of a sequence of Bernoulli trials: the probability of a Type 1 being p_1, and of a Type 2 being p_2. The reasons for distinguishing between the two driver types is that each type

has a distinct gap acceptance mechanism; typically a fast driver will accept or merge into a shorter main road gap than will a slow driver, so, conditional upon the driver type, his gap acceptance probability function will be given and he will behave in accordance with it. Putting this another way, a side road driver experiences interrupted service: he compares his critical gap to each new gap in main road traffic. In our present model he selects a new critical gap from his characteristic gap acceptance probability distribution for comparison to each new main road gap. When for the first time his critical gap is less than the main road gap he makes entry into the main road stream. Furthermore -- and this is a distinctive feature of this model -- the side road driver indeed uses the entirety of his critical gap while making entry. In actual fact, drivers perhaps "play safe", and wait for a main-road gap that is much longer than necessary for entry. However, if main road traffic is reasonably heavy and gaps typically short, drivers may well behave approximately as postulated here. If so, and if main-road traffic is Poisson, then instants of entry from the side road are regeneration points, and the side-road queue is an ordinary single-server system with independent completion times in place of service times. For simplicity we shall consider the service (completion) time to be made up entirely of the wait at the merge point (head of the queue) for a suitable gap to appear, and neglect the move-up time. This would seem reasonable under the conditions for which the present model is applicable.

GAP ACCEPTANCE AND COMPLETION TIMES

Our aim is to give numerical results for the model, so we have assumed a specific gap acceptance probability function for both "fast" and "slow" drivers. The probability density of the critical gap for a driver of Type i ($i=1$(fast), $=2$(slow)) is gamma:

$$f_i(x;\, m_i,\, k_i) = \left(\frac{t\, k_i}{m_i}\right)^{k_i - 1} \frac{1}{\Gamma(k_i)} \exp\left[-\frac{t\, k_i}{m_i}\right] \frac{k_i}{m_i}\,. \tag{1}$$

Since the expected value of the critical gap is m_i, and its variance is $m_i^2\, k_i^{-1}$, the critical gaps of a particular side-road driver may be forced to cluster as tightly as desired around any value for m_i by simply increasing k_i. We shall shortly illustrate the effect of changing the variance of the critical gap while its mean is held fixed.

The gap acceptance procedure is as follows. When a new driver moves into merge position, he is of type i with probability $p_i(i=1,2)$. He immediately selects a critical gap, G_1, from density f_i. If there are no main-road cars to arrive at the merger within time G_1 he enters, completing entry at time G_1 after he initially reached merge position. Suppose, however, that a main-road car will reach the entry within G_1. The side-road driver waits until the instant it passes the merge point, and then selects another, independent, gap, G_2, from (1), and behaves as he did with G_1, and so forth for G_3, etc. The probability that a side-road driver enters during any critical gap is the probability that the latter contains no main-road car, i.e.

$$\varphi_i(\nu) = \int_0^\infty e^{-\nu x} f_i(x;\, k_i,\, m_i)dx = \left(1 + \frac{\nu m_i}{k_i}\right)^{-k_i} \tag{2}$$

and the Laplace transform of the density function of a critical gap, given that entry is made, is

$$\frac{1}{\varphi_i(\nu)}\int_0^{\infty} e^{-sx}\, e^{-\nu x}\, f_i(x;\, k_i,\, m_i)dx = \frac{\varphi_i(\nu+s)}{\varphi_i(\nu)} \quad . \tag{3}$$

If G' denotes the above conditional critical gap, then $E[e^{-sG'}]$ is given by (3). The Laplace transform of the density function of a main road gap, given that it is shorter than the critical gap and hence no entry is made, is, putting $F_i(x)$ for the distribution function associated with (1),

$$\frac{1}{1-\varphi_i(\nu)} \cdot \int_0^{\infty} e^{-sx}\, [1-F_i(x)]e^{-\nu x}\nu\, dx = \frac{1-\varphi_i(s+\nu)}{1-\varphi_i(\nu)}\, \frac{\nu}{\nu+s} \,. \tag{4}$$

If M denotes such a conditional main-road gap, then $E[e^{-sM}]$ is given by (4). Finally

$$C = M_1 + M_2 + \dots + M_N + G' \tag{5}$$

where N denotes the number of too-short main road gaps; clearly

$$P\left\{N=n \,|\, \text{Type } i\right\} = [1-\varphi_i(\nu)]^n\, \varphi_i(\nu) \,. \qquad (n=0,\, 1,\, 2,\, \dots) \tag{6}$$

It then follows easily from the apparent independence that

$$\begin{aligned} E[e^{-sC_i}] &= \sum_{n=0}^{\infty} \left\{E[e^{-sM}]\right\}^n E[e^{-sG'}][1-\varphi_i(\nu)]^n\varphi_i(\nu) \\ &= \frac{(s+\nu)\varphi_i(s+\nu)}{s+\nu\varphi_i(s+\nu)} \quad . \end{aligned} \tag{7}$$

The Laplace transform of the completion time of a random driver attempting to merge is

$$E[e^{-sC}] = \sum_{i=1}^{2} p_i\, \frac{(s+\nu)\varphi_i(s+\nu)}{s+\nu\varphi_i(s+\nu)} \,, \tag{8}$$

and

$$E[C^2] = \sum_{i=1}^{2} 2p_i \left\{ \frac{1-\varphi_i(\nu)+\nu\varphi_i'(\nu)}{\nu^2\varphi_i^2(\nu)} \right\}, \tag{9}$$

$\varphi_i'(\nu)$ denotes the first derivative of φ_i at ν. Now the long-run expected waiting time for a side-road driver to enter can be obtained under our assumptions by using C as the service time in the classical formula:

$$\lim_{t\to\infty} E[W(t)] = \frac{\rho}{2(1-\rho)} \frac{E(C^2)}{E(C)}, \tag{10}$$

provided $\rho = \lambda E[C] < 1$; here $W(t)$ denotes waiting time at an instant t time units from process initiation.

The expression (10) is limited to assessing expected waits "a long time" after the process begins, and then only when $\rho < 1$. Thus it is of no direct use for predicting delays at any specific short time after some initial instant -- at which conditions, e.g. ν or λ, changed, as at the onset of rush hour -- or when the merging capacity is saturated, i.e. when $\rho > 1$. In order to exhibit the manner in which expected waiting time depends upon elapsed time, t, and also upon the initial state of the process, one way to proceed is to numerically invert the Laplace transform of the expected waiting time. Specifically, letting $N(t)$ denote the number of cars queued at the side road at time t, we invert

$$p(s) = \int_0^\infty e^{-st} E[W(t) \mid N(0) = i]dt, \tag{11}$$

a formula for which may be derived from general queueing theory. This formula is presented in [7], where a method of numerical transform inversion is also developed. The methods of [7] have been applied to several specific cases to obtain the numbers in the following table.

EXPECTED DELAY AT SIDE ROAD
(initially, no queue)

$p_1 = 0.5, \quad p_2 = 0.5$

$m_1 = 5, \quad m_2 = 15$ (secs)

$k_1 = k_2 = 2$

Case	τ(secs)	60 (secs)	120	180	240	600	900	Long-Run (10)
I	$\lambda = 0.05$, $\nu = 0.10$, $E(S) = 13.12$, $\rho = 0.656$	(14.6	19.5	22.2	24.0	27.7	28.3)	(28.7)
		27.7	32.6	35.3	37.1	40.8	41.4	41.8
II	$\lambda = 0.05$, $\nu = 0.15$, $E(S) = 14.69$, $\rho = 0.735$	(18.4	25.8	30.5	33.9	43.1	45.9)	(49.1)
		32.1	40.5	45.2	48.6	57.8	60.6	63.8
III	$\lambda = 0.075$, $\nu = 0.10$, $E(S) = 13.12$, $\rho = 0.984$	(25.7	39.0	49.2	57.7	94.1	115.8)	(946.9)
		38.8	52.1	62.3	70.8	107.2	128.9	960.0
IV	$\lambda = 0.075$, $\nu = 0.15$, $E(S) = 14.69$, $\rho = 1.102$	(31.9	50.4	65.6	78.9	143.2	187.9)	Not Applicable
		46.6	65.1	80.3	93.6	157.9	202.6	

$p_1 = 0.25$, $p_2 = 0.75$ (otherwise same as above)

Case	τ(secs)	60 (secs)	120	180	240	600	900	Long-Run (10)
V	$\lambda = 0.05$, $\nu = 0.10$, $E(S) = 16.88$, $\rho = 0.844$	(22.1	32.0	38.8	44.0	61.9	69.7)	(90.6)
		39.0	48.9	55.7	60.9	78.8	86.6	107.5

$p_1 = 0.75$, $p_2 = 0.25$ (otherwise same as above)

Case	τ(secs)	60 (secs)	120	180	240	600	900	Long-Run (10)
VI	$\lambda = 0.05$, $\nu = 0.10$, $E(S) = 9.38$, $\rho = 0.469$	(7.8	9.4	10.0	10.2	10.5	10.5)	(10.5)
		17.2	18.8	19.4	19.6	19.9	19.9	19.9

EXPECTED DELAY AT SIDE ROAD(Continued)
(initially, no queue)
$p_1 = 0.5, \quad p_2 = 0.5$
$k_1 = k_2 = 6$ (otherwise same as above)

Case \ τ(secs)		60 (secs)	120	180	240	600	900	Long-Run (10)
VII	λ = 0.05 ν = 0.10 E(S) = 17.15 ρ = 0.858	(23.3 40.5	34.2 51.4	41.9 59.1	48.0 65.2	69.5 86.7	79.5) 97.7	(112.7) 129.9
VIII	λ = 0.075 ν = 0.10 E(S) = 17.15 ρ = 1.29	(40.4 57.6	66.7 83.9	89.8 107.0	112.2 129.4	226.6 143.8	316.6)	Not Applicable

NOTE: Numbers in parentheses indicate expected wait until head of line is reached; other numbers indicate total time to make entry. Dimensions of λ and ν are [$secs^{-1}$], and of E(S) are [secs].

Although Table 1 was computed for specific parameter values, it helps to provide an understanding of the approximation afforded by the long-run formula, (10), when the latter is applicable. For comparison, the long-run values of expected waiting time corresponding to the various cases appear in the right-most column of the table. For example,

1) Comparing Case I to Case VII suggests the important effect of the "shape" or "sharpness" of the gap acceptance probability function, as measured by the parameter k, both upon the long-run expected wait and upon the rate at which the long-run values is approached. Although the expected critical gaps for both driver types are the same in both cases, in Case VII the probability that a side-road driver has a small critical gap is much smaller than in Case I. In the present model this is because the variance of the critical gaps for Case VII is one-third that in Case I. The result is that entry is more difficult, and the traffic intensity is larger, in Case VII, and the long-run value is much less rapidly approached.

2) Comparing Cases I, V, and VI, the effect of driver type mix is exhibited. Clearly a mixture of 25 percent slow-75 percent fast results in a drastically reduced wait, especially when compared to 75 percent slow-25 percent fast. Moreover, approach to the long-run value is correspondingly effected.

3) Finally, Cases III and VIII indicate the effect of the gap acceptance probability shape upon wait when the merge is close to saturation (former case) or over saturation (latter).

APPROXIMATIONS

Examination of expressions (5), (6), and (7) reveals that C_1 is a geometrically compounded random variable. If $E[C_1]$ becomes large, either because main-road traffic density is high, or the probability of an entry from the side-road, $\varphi_1(v)$, is small, then the distribution of C_1 is approximately exponential:

$$P\left\{C_1 \leq x\right\} \approx 1 - \exp\left\{ -x/E[C_1] \right\} \qquad (x \geq 0). \tag{12}$$

Tendency to the exponential in distribution can be shown formally by means of a continuity theorem for Laplace transforms.

The fact that the service-time distribution tends to the exponential suggests that an approximating process can be constructed, having exponential service times in place of the true service times of (8), that mimics closely both the transient and long-run behavior of the original process. It is this, and allied, approximations that we shall study numerically here.

Suppose $\left\{W(t),\ t \geq 0\right\}$ represents the original, or "true", delay process at the merge point; this process has arrival rate λ and completion times described by (8). In the light of (12), a first candidate for an approximate process is $\left\{W_m(t)\right\}$, where the latter has arrival rate λ and mixed exponential completion times:

$$E[e^{-sC_m}] = \frac{p_1}{1+sE[C_1]} + \frac{p_2}{1+sE[C_2]} \quad . \tag{13}$$

A second candidate is $\left\{W_s(t)\right\}$, differing from $\left\{W(t)\right\}$ only in replacing (13) by the single exponential

$$E[e^{-sC_s}] = \frac{1}{1+s\left\{p_1E[C_1] + p_2E[C_2]\right\}} \quad . \tag{14}$$

The degree of approximation thus obtained is illustrated in the next table for two cases.

Table 2

COMPARISON OF APPROXIMATIONS
FOR EXPECTED DELAY
(no initial queue)

$\lambda = 0.05$, $\nu = 0.10$; $m_1 = 5$, $m_2 = 15$

$k_1 = k_2 = 2$; $E[C_1] = 5.625$, $E[C_2] = 20.625$

I: $p_1 = 0.75$, $p_2 = 0.25$; $E[C] = 9.375$

$\rho = 0.469$

Case\τ(secs)	60	120	180	240	600	900
$E[W(\tau)]$	7.8	9.4	10.0	10.2	10.5	10.5
$E[W_m(\tau)]$	8.6	10.6	11.4	11.8	12.2	12.2
$E[W_s(\tau)]$	6.9	7.8	8.1	8.2	8.3	8.3

II: $p_1 = 0.25$, $p_2 = 0.75$; $E[C] = 16.875$

$\rho = 0.844$

Case\τ(secs)	60	120	180	240	600	900
$E[W(\tau)]$	22.1	32.0	38.8	44.0	61.9	69.7
$E[W_m(\tau)]$	23.3	34.2	41.7	47.6	68.1	77.3
$E[W_s(\tau)]$	22.3	32.1	39.0	44.2	62.2	70.1

Apparently, the approximation is only fair in I and respectable in II, with the single exponential approximation superior to the mixed exponential in II.

It is noticeable in I that neither approximation is approaching the limiting value for the true process. Unless main-road traffic is very dense ($\nu \to \infty$) the exponential (12) will not be precise, so we cannot in general expect the approximating processes to possess the same long-run limits as does $\{W(t)\}$. The latter limits depend upon higher moments than the first; parenthetically the close agreement of $E[W_s(t)]$ and $E[W(t)]$ in II above can be explained by the fact that the second moments of the respective completion times are very close numerically.

The source of an improved approximation is, thus, to force the approximating process to have the same long-run mean values as $\{W(t)\}$ as well as the same traffic intensity. Consider $\{W_d(t)\}$, having arrival rate λ_d and exponential completion time C_d such that

$$\rho = \lambda E[C] = \lambda_d E[C_d] = \rho_d \tag{15}$$

and

$$E[W] = \frac{\rho}{2(1-\rho)} \frac{E[C^2]}{E[C]} = \frac{\rho_d}{1-\rho_d} E[C_d] = E[W_d] . \tag{16}$$

Thus we put

$$\lambda_d = \lambda\left\{2\,\frac{E^2[C]}{E[C^2]}\right\} \tag{17}$$

$$E[C_d] = \frac{E[C^2]}{2E[C]} . \tag{18}$$

Another way of arriving at the approximation $\{W_d(t)\}$ is to argue in a manner reminiscent of that used to obtain the diffusion approximation to discontinuous processes; see Bailey [1] or Khintchine [10]. Consider the increments

$$\begin{aligned} \Delta W(t) &= W(t+\Delta t) - W(t) \\ \Delta W_d(t) &= W_d(t+\Delta t) - W_d(t) . \end{aligned} \tag{19}$$

We expect the development of $W(t)$ to be probabilistically similar to that of $W_d(t)$ if the distributions of the independent increments are similar. To bring about this similarity, equate the infinitesimal means and variances of the two processes; neglecting the effects of the boundary at zero we have

$$\lim_{\Delta t \to 0} E\left[\frac{\Delta W(t)}{\Delta t}\right] = \lambda E[C] - 1 \tag{20}$$

$$\lim_{\Delta t \to 0} E\left[\frac{\Delta W(t)}{\Delta t}\right] = \lambda_d E[C_d] - 1 \tag{21}$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \operatorname{Var}[\Delta W(t)] = \lambda E[C^2] \tag{22}$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \operatorname{Var}[\Delta W(t)] = \lambda_d E[C_d{}^2] . \tag{23}$$

Now equating traffic intensities in the two processes is precisely equivalent to equating the infinitesimal means or "drifts" of (20) and (21). Then if we specialize C_d to have the exponential distribution, and equate (22) and (23) the expressions (17) and (18) fall out.

The qualities of the two approximations suggested are illustrated in the following table.

Table 3

COMPARISON OF APPROXIMATIONS FOR EXPECTED DELAY

(No initial queue)

$p_1 = p_2 = 0.5$ $\quad$ $\lambda = 0.015$, $\nu = 0.25$; $E[C] = 39.051$

$m_1 = 5$, $m_2 = 15$ $\quad$ $\lambda_d = 0.01016$, $E[C_d] = 57.665$

$k_1 = k_2 = 2$

Case \ τ(secs)	60	130	180	240	600	900	1200
$E[W(\tau)]$	22.45	34.81	43.09	49.17	67.23	73.31	76.54
$E[W_s(\tau)]$	20.41	29.68	35.41	39.38	49.80	52.62	53.89
$E[W_d(\tau)]$	23.44	35.91	44.15	50.16	67.84	73.73	76.83

Although the approximation afforded by $\{W_s(t)\}$ is inadequate, that given by $\{W_d(t)\}$ would probably be entirely satisfactor for applications.

At present it is not known how well a direct diffusion approximation to $\{W(t)\}$ compares in quality to the exponential approximation $\{W_d(t)\}$. The diffusion approximation can be obtained, in principle, by solving the forward partial differential equation of diffusion theory for the density of waiting time, and then integrating to find the mean. Neither is it yet known how closely higher moments, e.g. the variance, of $\{W_d(t)\}$ conform to corresponding moments of $\{W(t)\}$, nor how well the probabilities of waits of various durations in the two processes agree. J.F.C. Kingman's heavy traffic theory [11], [12] does suggest that, in case side-road arrival rate, λ, increases so that traffic intensity approaches unity from below, the diffusion approximation becomes increasingly accurate. An increase in main-road density, ν, has a similar effect.

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13. ABSTRACT

Time-dep. delays at traffic merger

The expected delay of a side-road driver attempting to merge with, or cross, a main-road traffic stream is studied. The model includes the effect of mixture of "slow" and "fast" drivers at the side road, and of different gap acceptance probabilities. Numerical results show the manner in which long-run delays are approached, and an approximation to the transient behavior of delays is studied.

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